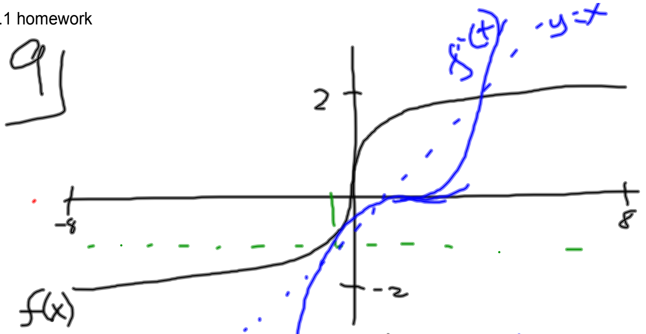
9)



a) f has an inverse b/c f(x) passes the horizontal line test; it is one to one.

$f^{-1}(2) = 8$

$f^{-1}(-1) = -1$

$f^{-1}(0) = 0$

Domain: [-8,8]

Range: [-2,2]

b) domain & range of $f^{-1}(x)$

domain of f(x) = range of $f^{-1}(x)$,
range of f(x) = domain of $f^{-1}(x)$

$f^{-1}(x)$: range = [-8,8] domain = [-2,2]

c) sketch graph of $f^{-1}(x)$...

Note: graph will be a reflection (across y=x) of f(x)

Packet #1

$f(2) = 2^3 + 2(2) - 1 = 11$

$f'(x) = 3x^2 + 2 \qquad f'(2) = 14$

$$\underbrace{f(x) = x^3 + 2x - 1, \quad x = 2}$$

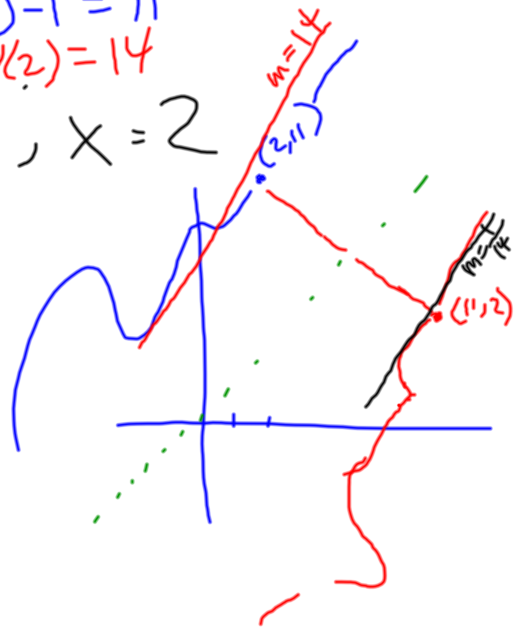
$$x = y^3 + 2y - 1$$

$$1 = 3y^2\left(\frac{dy}{dx}\right) + 2\left(\frac{dy}{dx}\right)$$

$$1 = \left(\frac{dy}{dx}\right)\left(3y^2 + 2\right)$$

$$\frac{d}{dx}\left(f^{-1}(x)\right) = \frac{dy}{dx} = \frac{1}{3y^2 + 2}$$

$$\left.\frac{dy}{dx}\right|_{\substack{f(2)=11 \\ f^{-1}(11)=2 \\ y=2}} = \frac{1}{3(2)^2 + 2} = \frac{1}{14}$$



Packet #3 $f(x)=\sin x;\ -\frac{\pi}{2}\le x\le\frac{\pi}{2},\ x=\frac{1}{2}$

Method 1 $f(\frac{1}{2})=\sin(\frac{1}{2})$

$f:(\frac{1}{2}, \sin(\frac{1}{2}))$ .878

$f^{-1}:(\sin(\frac{1}{2}), \frac{1}{2})$

$f^{-1}:(.878, \frac{1}{2})$

$y=\sin x$

$x=\sin y$

$\sin^{-1}(x)=y$ ☆

$\frac{d}{dx}(\sin^{-1}(x))=\frac{dy}{dx}$

$\sin^2+\cos^2=1$

$1-\sin^2=\cos^2$

$\frac{1}{\sqrt{1-x^2}}=\frac{dy}{dx}$

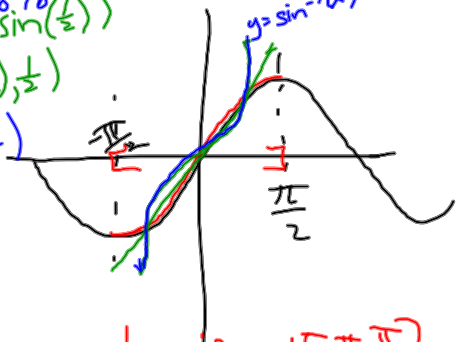
$\frac{1}{\sqrt{\frac{3}{4}}}=\frac{2}{\sqrt{3}}=\frac{2\sqrt{3}}{3}$

$\frac{3.464}{3}=1.155$

$\frac{1}{\sqrt{1-\sin^2(\frac{1}{2})}}=\frac{dy}{dx}$

$=\frac{1}{\sqrt{\cos^2\frac{1}{2}}}=\frac{dy}{dx}$

$1.139=\frac{1}{\cos\frac{1}{2}}=\frac{dy}{dy}$



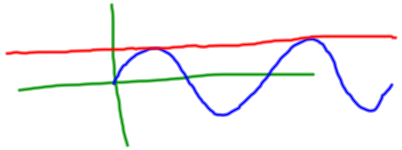
domain $\sin(x)$: $[-\frac{\pi}{2}, \frac{\pi}{2}]$

range $\sin x$: $[-1, 1]$

domain $\sin^{-1}(x)$: $[-1, 1]$

range $\sin^{-1}(x)$: $[-\frac{\pi}{2}, \frac{\pi}{2}]$

$$\sin^{-1}(x) = y$$



<u>Means</u>

"the angle whose sine is x" $= y$

$\sin^{-1}(1) = \frac{\pi}{2}$ $\quad$ $\begin{cases} \sin(x) = 1 \\ x = \frac{\pi}{2} \pm n(2\pi) \end{cases}$

$\sin^{-1}(0) = 0$

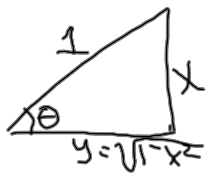
$\sin^{-1}(-1) = -\frac{\pi}{2}$

$f(f^{-1}(x)) \doteq x$

$\sin(\sin^{-1}(x)) = x$ $\quad$ apply for $x \in [-1, 1]$

$\cos(\sin^{-1}(x)) \cdot [\sin^{-1}(x)]' = 1$

$$[\sin^{-1}(x)]' = \frac{1}{\cos(\sin^{-1}(x))} = \frac{1}{\cos(\theta)} = \frac{1}{\sqrt{1-x^2}}$$



$1^2 = x^2 + y^2$

$y = \sqrt{1-x^2}$

Packet #3 $f(x) = \sin x;\ -\frac{\pi}{2} \le x \le \frac{\pi}{2},\ x = \frac{1}{2}$

$(\frac{1}{2}, \sin(\frac{1}{2}))$

$f^{-1}(\sin(\frac{1}{2}), \frac{1}{2})$

Method 2

$x = \sin(y)$

Diff:

$1 = \cos(y)\frac{dy}{dx}$

$\frac{1}{\cos(\frac{1}{2})} = \frac{1}{\cos(y)} = \frac{dy}{dx}$

1.139

$, \frac{1}{2}]$

5) $f(x) = x^3 - \frac{4}{x}$; $x > 0$; $x = 6$ $f: \left(6, \frac{646}{3}\right)$

$f(6) = 6^3 - \frac{4}{6} = 215\frac{1}{3} = \frac{646}{3}$ $f^{-1}: \left(\frac{646}{3}, 6\right)$

$x = y^3 - \frac{4}{y}$

$\frac{d}{dx}\left(-4y^{-1}\right) = +4y^{-2}\frac{dy}{dx}$

Diff:

$1 = 3y^2\frac{dy}{dx} + \frac{4}{y^2}\frac{dy}{dx}$

$\frac{d}{dx}\left(\frac{-4}{y}\right) = \frac{(0)y - (-4)\left(\frac{dy}{dx}\right)}{y^2}$

$\frac{d}{dx}\left(\frac{f}{g}\right) = \frac{f'g - fg'}{g^2}$

$1 = \frac{dy}{dx}\left(3y^2 + \frac{4}{y^2}\right)$

$\frac{1}{3y^2 + \frac{4}{y^2}} = \frac{dy}{dx}$

$\left.\frac{dy}{dx}\right|_{y=6} = \frac{1}{3(6)^2 + \frac{4}{6^2}} = \frac{1}{108 + \frac{1}{9}} = \frac{9}{973}$

~~4.1 homework~~

2010-11-05 Pd 2

3.6 examples

$x^2y - x\sin y = 3$

Diff w.r.t. x

$\left[(2x)(y) + (x^2)\left(\frac{dy}{dx}\right)\right] - \left[(1)\sin y + (x)(\cos y)\frac{dy}{dx}\right] = 0$

$2xy + x^2\frac{dy}{dx} - \sin y - x\cos y\frac{dy}{dx} = 0$

$\frac{dy}{dx}(x^2 - x\cos y) = \sin y - 2xy$

$\frac{dy}{dx} = \frac{\sin y - 2xy}{x^2 - x\cos y}$

3.6 Implicit Differentiation

define a function explicitly by telling me what $y =$

define a function implicitly by telling me a relationship satisfied by x and y.

$(2, f(2))$

$(2, 11) \neq \boxed{x = 2}$

$f^{-1}: (11, 2)$

$$f(x) = x^3 + 2x - 1$$

$f'(x) = 3x^2 + 2$

$f'(2) = 14$

Replace y w/ x

$$(x)' = [y^3 + 2y - 1]'$$

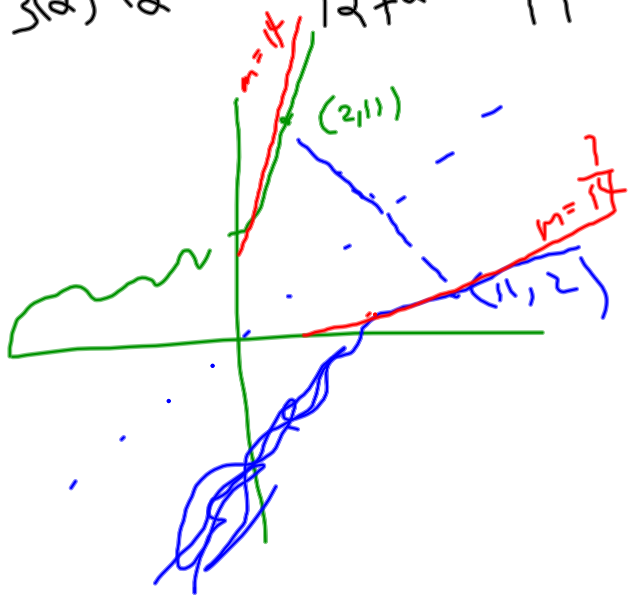
$$1 = [3y^2 + 2]\frac{dy}{dx}$$

$$\frac{1}{[3y^2 + 2]} = \frac{dy}{dx}$$

$$\frac{1}{3(2^2) + 2} = \frac{1}{12 + 2} = \frac{1}{14}$$

$$[y^3 + 2y - 1]' = \left(3y^2\frac{dy}{dx}\right) + 2\frac{dy}{dx}$$

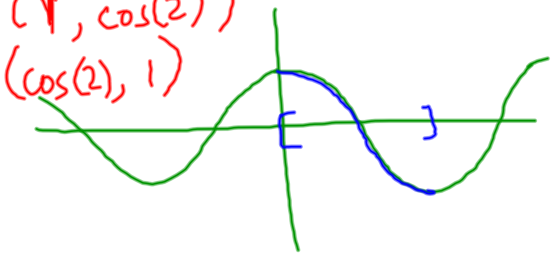
$$= \frac{dy}{dx}(3y^2 + 2)$$



4| $f(x) = \cos(2x)$; $\cancel{-\frac{\pi}{2} \le x \le \frac{\pi}{2}}$ $0 \le x \le \pi$ at $x = 1$

$(1, \cos(2))$

$(\cos(2), 1)$



Method I

$$x = \cos(2y)$$

$$\cos^{-1}(x) = 2y$$

$$\frac{1}{2}\cos^{-1}(x) = y$$

$$\frac{1}{2}\left(\frac{-1}{\sqrt{1-x^2}}\right) = \frac{dy}{dx}$$

$$\frac{-1}{2\sqrt{1-\cos^2 2}} = \frac{dy}{dx}$$

$$\frac{-1}{2\sqrt{\sin^2 2}} =$$

$$\frac{-1}{2\sin 2} =$$

$$\boxed{\begin{array}{l} \sin^2 + \cos^2 = 1 \\ \cancel{\cos} \\ \sin^2 = 1 - \cos^2 \end{array}}$$

domain $\cos(\theta)$: $[0, \pi]$

range $\cos\theta$ $[-1, 1]$

domain $\cos^{-1}(\theta)$ $[-1, 1]$

range $\cos^{-1}(\theta)$ $[0, \pi]$

$$\cos^{-1}(x) = y$$

"the angle whose cosine is x" $= y$

$$\cos^{-1}(1) = 0$$

$$\cos(x) = 1$$

$$0 + (2n\pi)$$

$$\cos^{-1}(0) = \frac{\pi}{2}$$

$$\cos^{-1}(-1) = \pi$$

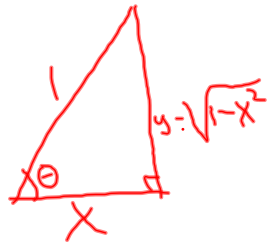
$$f(f^{-1}(x)) = x$$

$$\cos(\cos^{-1}(x)) = x$$

for every x in domain of $\cos^{-1}(x)$

$$-\sin(\cos^{-1}(x)) \cdot [\cos^{-1}(x)]' = 1$$

$$[\cos^{-1}(x)]' = \frac{-1}{\sin(\cos^{-1}(x))}$$



$$1^2 = x^2 + y^2$$

$$= \frac{-1}{\sin\theta} = \frac{-1}{\sqrt{1-x^2}}$$

$f(x) = \cos(2x)$; $0 \le x \le \pi$; $x = 1$

$(1, \cos 2)$

$(\cos 2, 1)$

Method 2

$x = \cos(2y)$

$1 = -\sin(2y)\left(2\frac{dy}{dx}\right)$

$\frac{-1}{2\sin(2y)} = \frac{dy}{dx}$

$\frac{-1}{2\sin 2} =$

#6 $f(x) = \sqrt{x-4}$; $x = 2$

$f^{-1}: (8, 2)$

$\sqrt{x-4} = 2$

$x - 4 = 4$

$x = 8$

domain: $[4, \infty)$ (2, does not exist)

range $[0, \infty)$ (does not exist, 2^{4}) exist

$x = \sqrt{y-4}$

$x^2 = y - 4$

$x^2 + 4 = y$ → $\frac{dy}{dx} = 2x$